

Test Assignment #2

1) Given $R(q) = 16q - 0.08q^2$ and $C(q) = 30 + 10q - 0.02q^2$

$$\begin{aligned} \text{(i) } MC(q) &= \lim_{h \rightarrow 0} \frac{C(q+h) - C(q)}{h} \\ &= \lim_{h \rightarrow 0} \frac{[30 + 10(q+h) - 0.02(q+h)^2] - [30 + 10q - 0.02q^2]}{h} \\ &= \lim_{h \rightarrow 0} \frac{30 + 10q + 10h - 0.02(q^2 + 2qh + h^2) - 30 - 10q + 0.02q^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{10h - 0.02q^2 - 0.04qh - 0.02h^2 + 0.02q^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{10h - 0.04qh - 0.02h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(10 - 0.04q - 0.02h)}{h} \\ &= \lim_{h \rightarrow 0} (10 - 0.04q - 0.02h) \\ &= 10 - 0.04q - 0.02(0) = 10 - 0.04q \end{aligned}$$

$$\begin{aligned} MR(q) &= \lim_{h \rightarrow 0} \frac{R(q+h) - R(q)}{h} \\ &= \lim_{h \rightarrow 0} \frac{[16(q+h) - 0.08(q+h)^2] - [16q - 0.08q^2]}{h} \\ &= \lim_{h \rightarrow 0} \frac{16q + 16h - 0.08(q^2 + 2qh + h^2) - 16q - 0.08q^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{16h - 0.08q^2 - 0.16qh - 0.08h^2 - 0.08q^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{16h - 0.16qh - 0.08h^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(16 - 0.16q - 0.08h)}{h} \\ &= \lim_{h \rightarrow 0} (16 - 0.16q - 0.08h) \\ &= 16 - 0.16q - 0.08(0) = 16 - 0.16q \end{aligned}$$

(11) When $MC(q) = MR(q)$

$$\Rightarrow 10 - 0.04q = 16 - 0.16q$$

$$\Rightarrow -0.04q + 0.16q = 16 - 10$$

$$\Rightarrow 0.12q = 6$$

$$\Rightarrow q = \frac{6}{0.12} = 50 \text{ will yield the max profit}$$

$$\begin{aligned} \text{Profit, } P &= R - C = (16q - 0.08q^2) - (30 + 10q - 0.02q^2) \\ &= 16q - 0.08q^2 - 30 - 10q + 0.02q^2 \\ &= -0.06q^2 + 6q - 30 ; q \geq 0 \end{aligned}$$

when $q = 50$,

$$\begin{aligned} P &= -0.06(50^2) + 6(50) - 30 \\ &= \$120 \text{ is the max profit} \end{aligned}$$

2) (i) $\lim_{x \rightarrow -2} \frac{x^2 - 4}{x^2 - x - 6} = \lim_{x \rightarrow -2} \frac{(x-2)(x+2)}{(x+2)(x-3)}$

$$= \lim_{x \rightarrow -2} \frac{x-2}{x-3} = \frac{-2-2}{-2-3} = \frac{-4}{-5} = \frac{4}{5}$$

(ii) $\lim_{h \rightarrow 0} \frac{\sqrt{4+h} - 2}{h} = \lim_{h \rightarrow 0} \left(\frac{\sqrt{4+h} - 2}{h} \cdot \frac{\sqrt{4+h} + 2}{\sqrt{4+h} + 2} \right) = \lim_{h \rightarrow 0} \frac{(\sqrt{4+h})^2 - 2^2}{h(\sqrt{4+h} + 2)}$

$$= \lim_{h \rightarrow 0} \frac{(4+h) - 4}{h(\sqrt{4+h} + 2)} = \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{4+h} + 2)}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{4+h} + 2} = \frac{1}{\sqrt{4+0} + 2} = \frac{1}{\sqrt{4} + 2} = \frac{1}{2+2} = \frac{1}{4}$$

(iii) $\lim_{x \rightarrow 2} \left(\frac{1}{x-2} - \frac{4}{x^2-4} \right) = \lim_{x \rightarrow 2} \left(\frac{1}{x-2} - \frac{4}{(x-2)(x+2)} \right)$

$$= \lim_{x \rightarrow 2} \left(\frac{x+2}{(x-2)(x+2)} - \frac{4}{(x-2)(x+2)} \right) = \lim_{x \rightarrow 2} \frac{x+2-4}{(x-2)(x+2)}$$

$$= \lim_{x \rightarrow 2} \frac{x-2}{(x-2)(x+2)} = \lim_{x \rightarrow 2} \frac{1}{x+2} = \frac{1}{2+2} = \frac{1}{4}$$

$$3) (i) \lim_{x \rightarrow -2^-} \frac{3x-2}{x+2} = \frac{3(-2)-2}{-2.0000...01+2}$$

$$= \frac{-8}{\text{-ve no. very close to zero}} = +\infty$$

$$\lim_{x \rightarrow -2^+} \frac{3x-2}{x+2} = \frac{3(-2)-2}{-1.9999...99+2}$$

$$= \frac{-8}{\text{+ve no. very close to zero}} = -\infty$$

$$(ii) \lim_{x \rightarrow -\infty} \frac{3x-2}{x+2} = \lim_{x \rightarrow -\infty} \frac{x(\frac{3x-2}{x})}{x(\frac{x+2}{x})} = \lim_{x \rightarrow -\infty} \frac{x(\frac{3x}{x} - \frac{2}{x})}{x(\frac{x}{x} + \frac{2}{x})}$$

$$= \lim_{x \rightarrow -\infty} \frac{x(3 - \frac{2}{x})}{x(1 + \frac{2}{x})} = \lim_{x \rightarrow -\infty} \frac{3 - \frac{2}{x}}{1 + \frac{2}{x}}$$

$$= \frac{3-0}{1+0} = 3$$

$$\lim_{x \rightarrow \infty} \frac{3x-2}{x+2} = \lim_{x \rightarrow \infty} \frac{x(3 - \frac{2}{x})}{x(1 + \frac{2}{x})} = \lim_{x \rightarrow \infty} \frac{3 - \frac{2}{x}}{1 + \frac{2}{x}}$$

$$= \frac{3-0}{1+0} = 3$$

(iii) There is a vertical asymptote at $x = -2$
and a horizontal asymptote at $y = 3$

4) Given $y = \begin{cases} cx^2 + cx + 1, & x \leq 2 \\ 2cx + 2, & x > 2 \end{cases}$

For this function to be continuous at $x=2$, we need

$$\lim_{x \rightarrow 2^-} y = \lim_{x \rightarrow 2^+} y = y(2)$$

$$\Rightarrow \lim_{x \rightarrow 2^-} (cx^2 + cx + 1) = \lim_{x \rightarrow 2^+} (2cx + 2) = c(2^2) + c(2) + 1$$

$$\Rightarrow c(2^2) + c(2) + 1 = 2c(2) + 2 = c(2^2) + c(2) + 1$$

$$\Rightarrow 4c + 2c + 1 = 4c + 2$$

$$\Rightarrow 2c = 2 - 1 = 1$$

$$\Rightarrow c = \frac{1}{2}$$